Roll No.							

24-MA-24

M.Sc. II SEMESTER [MAIN/ATKT] EXAMINATION
JUNE - JULY 2024

MATHEMATICS
Paper - IV
[Complex Analysis - II]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt five questions. Each question carries an internal choice.
Each question carries **15 marks.**

Q. 1 State and Prove Arzela - Ascoli Theorem. **(15 Marks)**

OR

State and Prove Montel's Theorem. **(15 Marks)**

Q. 2 State and prove Legendre's duplication formula. **(15 Marks)**

OR

State and prove Riemann's functional equation. **(15 Marks)**

Q. 3 If f(z) is analytic within and on the circle γ such that $|z| = R$, and if f(z) has zeros at the points $a_i \neq 0$ (i = 1, 2, . . . , m) and poles at $b_j \neq 0$, (j = 1, 2, . . . , n) inside γ, multiple zeros and poles being repeated, then prove that **(15 Marks)**

$$\frac{1}{2\pi}\int_0^{2\pi} \log|f(Re^{i\theta})|\,d\theta \quad \log|f(0)| = \sum_{i=1}^{m} \log\frac{R}{|a_i|} - \sum_{j=1}^{n} \log\frac{R}{|b_j|}$$

OR

a) Prove that the convergence σ of a sequence $\{Z_n\}$ is given by - **(08 Marks)**

$$\sigma = \lim_{n \to \infty} \sup \frac{\log n}{\log |Z_n|}$$

b) If f(z) is an entire function of order ρ and convergence exponent σ, then prove that $\sigma \leq \rho$ **(07 Marks)**

Q. 4 a) State and prove Schwartz's reflection principle. **(10 Marks)**

b) Prove that there cannot be more than one analytic continuation of a function into the same domain. **(05 Marks)**

OR

Let $\gamma : \rightarrow \mathbb{C}$ [0, 1] be a path from a to b and let $\{(f_t, D_t) : 0 \leq t \leq 1\}$ be an analytic continuation along γ. There is a number $\varepsilon > 0$ such that if $\sigma : [0, 1] \rightarrow \mathbb{C}$ is any path from a to b with $|\gamma(t) - \sigma(t)| < \varepsilon$ for all t and y $\{(g_t, B_t) : 0 \leq t \leq 1\}$ is any analytic continuation along σ with $[g_0]_a = [f_0]_a$; then prove that $[g_1]_b = [f_1]_b$ **(15 Marks)**

Q 5 a) If $u : \overline{B}(a, R) \rightarrow \mathbb{R}$ is continuous harmonic in B(a, R) and $u \geq 0$ then for $0 \leq r < R$ and all θ prove that **(10 Marks)**

$$\frac{R - r}{R + r} \; u(a) \leq u(a + r e^{i\theta}) \leq \frac{R + r}{R - r} \; u(a)$$

b) Prove that the poisson Kernel $P_r(\theta)$ can be expressed as - **(05 Marks)**

$$P_r(\theta) = \frac{1 - r^2}{1 - 2 r \cos\theta + r^2} = \text{Re}\left(\frac{1 + r e^{i\theta}}{1 - r e^{i\theta}}\right)$$

OR

Let $D = \{Z : |Z| < 1\}$ and suppose that $f : \partial D \rightarrow \mathbb{R}$ is a continuous function. Then prove that there is a continuous function $u : \overline{D} \rightarrow \mathbb{R}$ such that **(15 Marks)**

i) $u(z) = f(z)$ for z in ∂D.

ii) u is harmonic in D.

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